THE $\vec{E}$ – FIELD (COULOMB) / GRAVITATIONAL FIELD

FORCE BETWEEN TWO POINT CHARGES

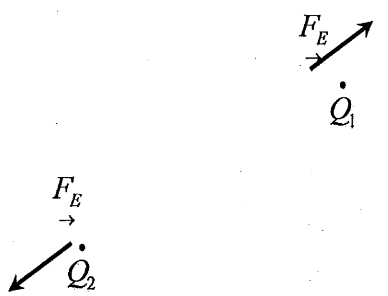
$$\vec{F}_E = k_e \frac{Q_1 Q_2}{r^2}\hat{r}$$

$$k_e = 9x10^9 \frac{N-m^2}{C^2}$$

Q_1, Q_2 *same sign*

$\vec{F}_E \parallel +\hat{r}$

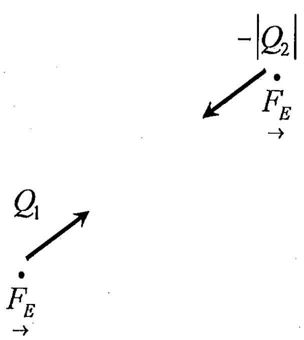
REPULSIVE (Both Forces Outward)



Q_1, Q_2 *have opposite signs.*

$\vec{F}_E \parallel -\hat{r}$ (Both Forces Inward)

ATTRACTIVE



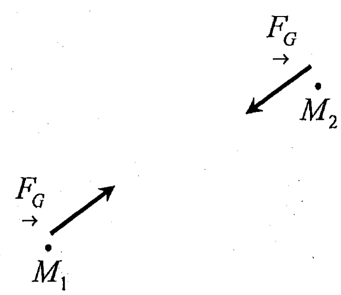
FORCE BETWEEN TWO POINT MASSES

$$\vec{F}_G = -\frac{GM_1M_2}{r^2}\hat{r}$$

$$G = 6.7x10^{-11}\frac{N-m^2}{(kg)^2}$$

$\vec{F}_G$ ALWAYS ATTRACTIVE

$\vec{F}_G \parallel -\hat{r}$ (Both Forces Inward)



NOTICE THAT THE FORCES OCCUR AS ACTION-REACTION PAIRS IN EVERY CASE.

$\left[\vec{F}_{12} = -\vec{F}_{21}\right]$ The Equations Represent Two Forces

Many Point Charges Force on Q_i

$$\underset{\rightarrow}{F_i} = k_e \frac{\Sigma}{j \neq i} \frac{Q_i Q_j}{r_{ij}^{2}} \hat{r}_{ij}$$

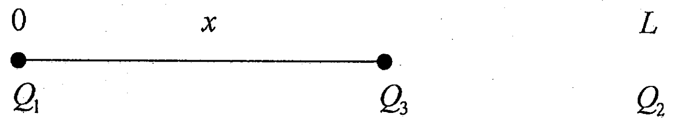
Note: Right side involves addition of vectors.

SPECIAL CASES

1. Q_1 at x=0, Q_2 at x=L. Where to locate Q_3 so $\underset{\rightarrow}{F_3}$ force on Q_3 is zero.

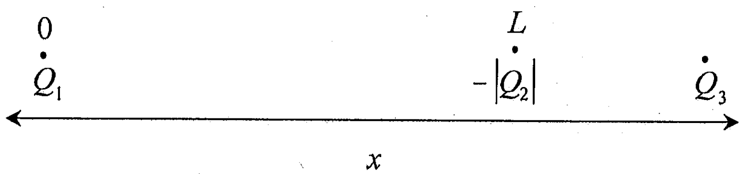
Q_3 must be on the line joining Q_1 *and* Q_2.

$$x = \frac{L}{1 + \sqrt{\frac{Q_2}{Q_1}}}$$

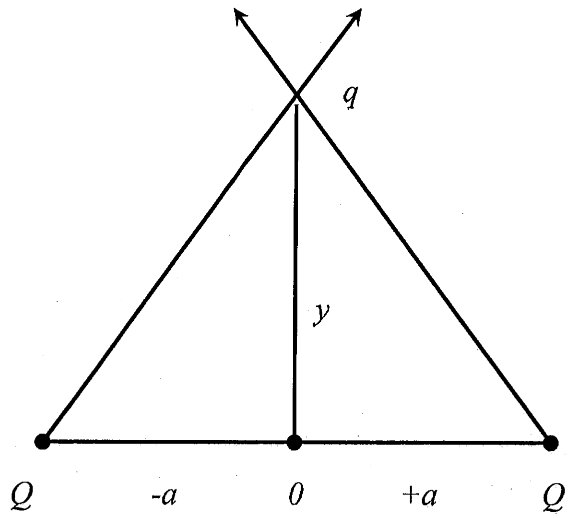


2. Q_1 at x=0, $-|Q_2|$ at x=L. $\underset{\rightarrow}{F_3}$ will be zero if

$$x = \frac{L}{\sqrt{\frac{Q_1}{Q_2}} - 1} \quad when\ Q_1 > |Q_2|$$



3. Q at x=-a, Q at x= +a. What is force on q at (0, y)



$$\underset{\rightarrow}{F_E}(y) = \frac{2k_e Q y q}{(y^2 + a^2)^{3/2}} \hat{y}$$

What if we have $-|q|\ at\,(0, y)$

$$\frac{-2k_e |q| Q y\, \hat{y}}{(y^2 + a^2)^{3/2}}$$

• $-|q|$

Let us make y << a, •– – – – – – – – · – – – – – – – –•

Q o Q

In this case, force is proportional to displacement y and opposite to it so $-|q|$ will show Linear Harmonic oscillations.

<u>QUESTION:</u> Why is there a force between two charges (masses) when they are far apart from one another?

To answer this we develop the concept of a <u>FIELD</u>

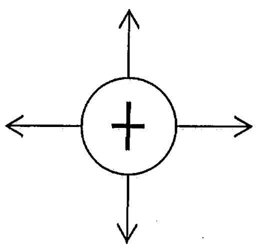
COULOMB $\underset{\rightarrow}{E}$*– FIELD*	*GRAVITATIONAL* $\underset{\rightarrow}{(G_F)}$ *FIELD*
If there is a charge Q sitting at x=0, the space around it is <u>not</u> empty. Q creates a coulomb $\underset{\rightarrow}{E}$ field which permeates all of space. If you place a test charge q in this $\underset{\rightarrow}{E}$ field, it experiences a force $\underset{\rightarrow}{F_E} = q\,\underset{\rightarrow}{E}$	If there is a mass M sitting at x=0, the space around it is not empty. M creates a gravitational $\underset{\rightarrow}{(G_F)}$ field which permeates all of space. If you place a test mass m in the $\underset{\rightarrow}{G}$ field it experiences a force $\underset{\rightarrow}{F_G} = m\underset{\rightarrow}{G_F}$

Formal Definitions: $\vec{G}_F$: If a stationary mass M feels a force and there is no visible agency applying the force, then M must be located in a gravitational field.

$\vec{E}$: If a stationary charge Q feels a force and there is no visible agency applying the force then Q must be located in an $\vec{E}$-field.

1. Single +ive charge Q at r=0.

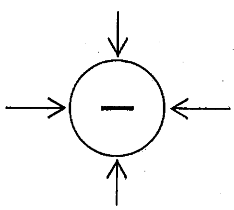
$$\vec{E} = \frac{k_e Q}{r^2}\hat{r}$$



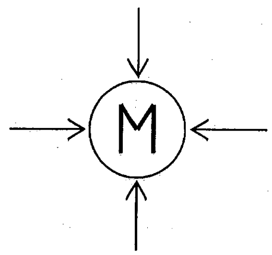
Acts like a "source" of an $\vec{E}$ field which points radially outward

2. Single –ive charge at r=0.

$$\vec{E} = -k_e \frac{|Q|}{r^2}\hat{r}$$



Acts like a "sink" of $\vec{E}$ field which points. Radially inward.

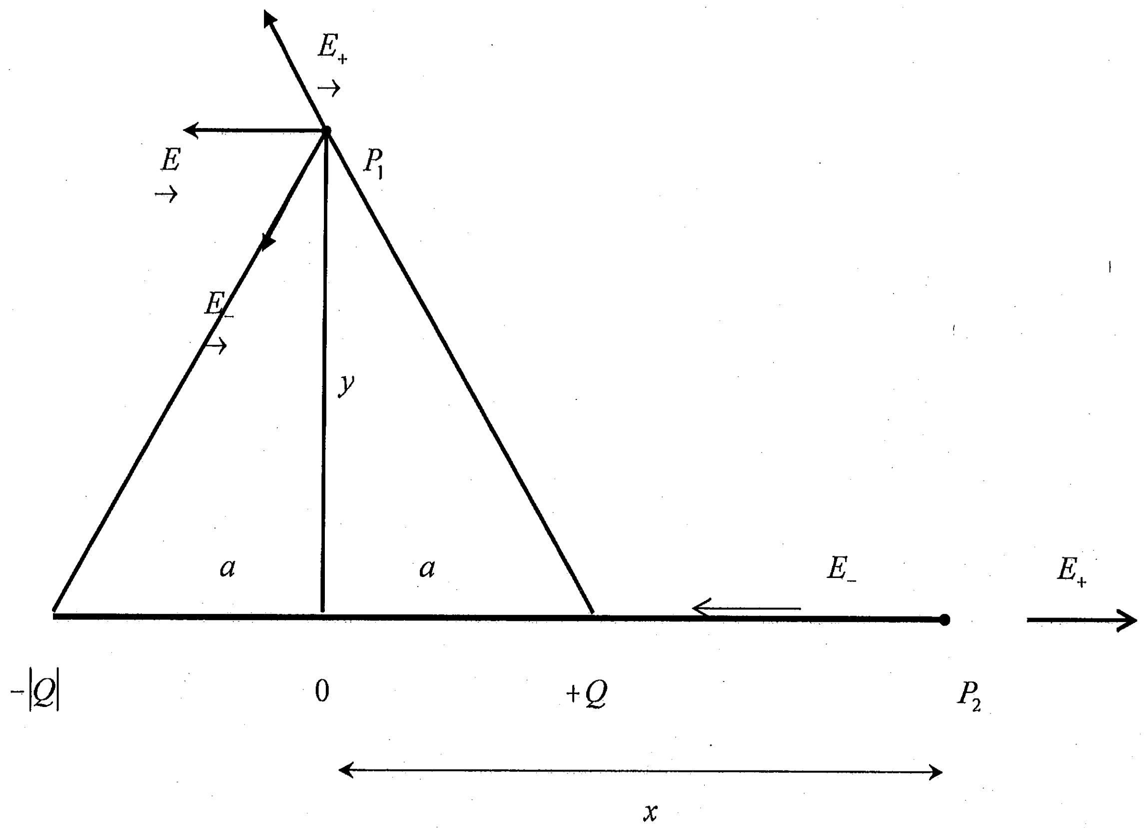


$$\vec{G}_F = -\frac{GM}{r^2}\hat{r}$$

ALWAYS INWARD RADIALLY.

3. $\underset{\to}{E}$ field of Dipole: $-|Q|$ *at x=-a*

$+Q$ *at* $x = a$



at $P_1 = (0, y)$ $\quad \underset{\to}{E}(y) = -\dfrac{k_e 2aQ}{(y^2 + a^2)^{3/2}}\hat{x}$

at $P_2 = (x, 0)$ $\quad \underset{\to}{E}(x) = -\dfrac{k_e 4aQx}{\left(x^2 - a^2\right)^2}$

Next, define dipole moment $\underset{\to}{p} = 2aQ\hat{x}$

$$\underset{\to}{E}(y) = \frac{-k_e \underset{\to}{p}}{y^3}$$

$$\underset{\to}{E}(x) = \frac{2k_e \underset{\to}{p}}{x^3}$$

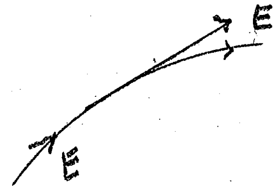
When *x,y* >>*a* that is far away from dipole

Properties of $\underset{\rightarrow}{E}$-Field Lines:

Following Faraday we can visualize an $\underset{\rightarrow}{E}$-field by drawing $\underset{\rightarrow}{E}$-field lines using the following rules.

1. $\underset{\rightarrow}{E}$-field lines can start or stop only at charges or head to infinity: start at positive charges, stop at negative charges.

2. The direction of the $\underset{\rightarrow}{E}$-field is given by the tangent to the field line.



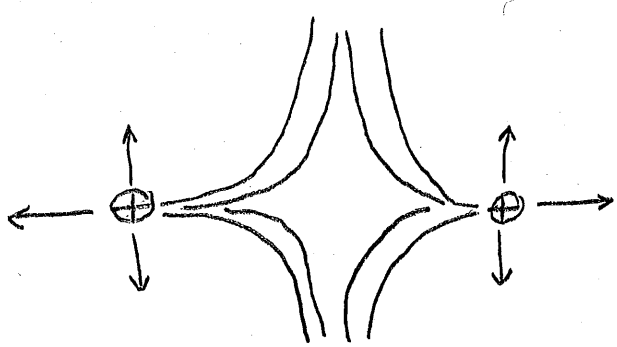
3. On account of 2, no two lines can intersect in space. Otherwise, the $\underset{\rightarrow}{E}$-field would have two directions at the same point.

4. The magnitude of $\underset{\rightarrow}{E}$ is proportional to the number of lines crossing a unit area perpendicular to the lines.

5. In drawing $\underset{\rightarrow}{E}$-field lines, the number of lines must be proportional to the charge.

$\underset{\rightarrow}{E}$-field lines

1) Two equal positive charges



2) Dipole: -Q and +Q